Lukass Kellijs handout

# General problem solving principles

**Olympiad level:**

1. If possible, **read all problems right at the start**, and plan your solving strategy
2. When solving problems – **use drafts to a minimum** – when starting a problem, evaluate whether to start it on an answer sheet.
3. **Writing out any and all of your ideas for solving the problem**, to the answer pages (Optimally, spend your last ~ 30 minutes doing this for problems, where this was not yet done).
4. Read the problem at the absolute minimum 2 times – the second time read it like you would read an SAT reading question – **analyze each word for scrupulously for its meaning**. (trust me it’s worth it - for not making mistakes like NBPhO 2022 pr.1 and NBPhO 2023 pr. 6)

**Problem level:**

1. Start solving your problem by **going in depth** into what actually goes on in the system, write down: what processes take place, what things should potentially be taken into account, what quantities might be conserved, what quantities are very small and can be neglected/approximated.
2. **Be creative**. Think about many different ways to explain or to view the processes in the problem (use creative analogies, hypothetical scenarios, limiting cases etc.)
3. **Write out all potential solving tactics/ideas** **right at the start** of the problem.
4. When pursuing a potential solution path, if you don’t see an end or the next 2 steps, **evaluate whether you’re not wasting your time**.
5. In general, **know when to stop and regroup**. Otherwise, you might really waste your time and/or get frustrated, which will badly effect other problem solving.
6. **Don’t be an idiot** – check everything.
7. **Be calm** – the more you try to hurry things up – the more time it takes – when thinking – slow and steady does it.

**Studying:**

1. Really understand every example problem. Reproduce the results yourself i.e. write the solution yourself using your own reasoning – this is incredibly important

**The key:**

1. Think physically. Understand the system and the processes within – completely. Don’t think through equations. (This is your biggest issue – you try to solve a problem for solving’s sake – to get points – which ends up harming you because you chaotically write down equations without considering alternative methods or physical reality) Take the time to understand. Then only use equations as a helping tool. To even come up with the creative solution you have to understand the problem. – **It is absolutely vital that you remember this.**

# Kinematics

Problems involving friction (air, ground etc.)

If friction affects the motion then usually the most appropriate reference frame (RF) is that of the environment causing the friction. In this environment the velocity vector doesn’t change direction and that is key.

We can transform into the Friction RF, express the velocity vector of the moving object, and then transform back, expressing the velocity vector of the object in the Ground RF as the vector sum of the velocity of the object in the Friction RF and the velocity of the Friction RF. This would give us a nice vectorial representation of the movement (example: pr 6. Kalda Kinematics).

Additionally, we can work with displacements in the Friction RF. Since we know the velocity vector, we know the direction of motion and therefore the direction of displacement. If we can then express the displacement itself (in the Friction RF), we can obtain the displacement in the ground RF if we sum vectorially the displacement in the Friction RF and that of the Friction RF itself (example: pr 7. Kalda Kinematics).

# Mechanics

# **ICOR**

**Diagram, engineering drawing

Description automatically generated**- instant center of rotation - the point fixed to a body undergoing planar movement that has **zero velocity** at a particular instant of time. At this instant, the velocity vectors of the other points in the body generate a circular field around this point which is identical to what is generated by a pure rotation.

This approach can also be very useful in determining frictional forces (i.e. finding symmetric circles upon which, when rotating about the ICOR the frictional forces cancel each other).

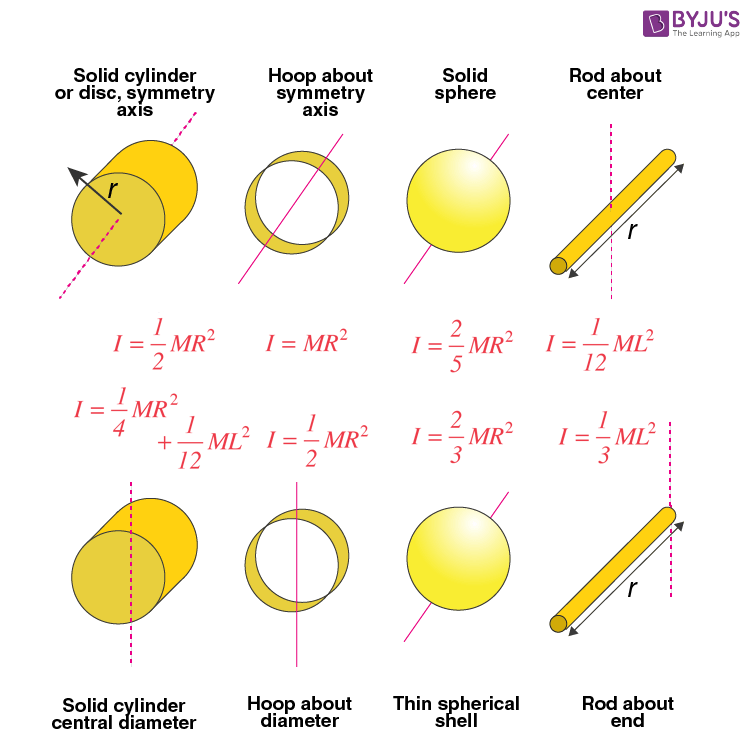
**Lagrangian Formalism – generalised coordinate approach**

Let us call a generalised coordinate if the entire state of a system can be described by this single number.

Say we need to find the acceleration of coordinate . If we can express the potential energy of the system as a function of and the kinetic energy in the form where coefficient M is a combination of masses of the bodies (and perhaps of moments of inertia), then

This generalised coordinate can be used in many ways along with different other powerful techniques, for example:

**The components of a net force** can be determined by finding the change of the location of the centre of mass of the system with respect to the generalised coordinate . By differentiating this via time, we get the acceleration of the centre of mass in each axis in terms of the acceleration of the generalised coordinate, and thus we can determine the components of the net force and hence its magnitude. (See Kalda handout pr 32.)

**Moments of inertia**

The thin spherical shell formula can be used to derive the moment of inertia of a non-uniform density sphere.

Moment of inertia contains important information about the geometry of the system and can even be used in reverse to find this geometry.

Moment of inertia with respect to the center of mass can be found also using considerations of the dynamics of the system:

, where L – angular momentum and Ek – is kinetic energy with respect to the center of mass **of a rigid body**. This equation is obtained through the analogy of linear and angular momentum, where the total kinetic energy is related with the total momentum. And it is useful in the CM refference frame because there Ek and L are conserved. (See pr. 1. EuPhO 2018)

# Orbītas, Keplera likumi

Pamata sakarības var iegūt sākot ar *Uniform Circular Motion* formulām pielīdzinot centrtieces spēku gravitācijas spēkam iegūstot:

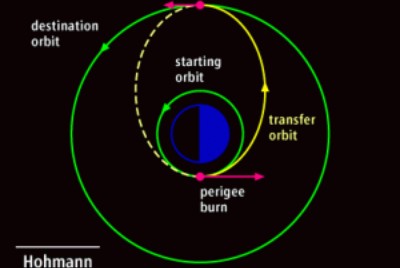
(speciālgadījums *Circular Orbits* – nav vispārīgi spēkā)

No kā izriet, ka

**Vispārīgo orbītu sakarības** ir ļoti līdzīgas, taču R aizvieto ar a – *semi-major axis length:*

Enerģija tiek saglabāta:

*Angular momentum* is conserved*:*

Varam secināt, ka mainās orbītas laikā, kā arī mainās orbītas laikā. Jāatceras arī, ka kādā punktā var iegūt, izmantojot enerģijas vienādojumu, pielīdzinot kopējo enerģiju 0.

**Orbītu maiņas:**

Visbiežāk sastopamās orbītu maiņas redzamas iepriekšējā attēlā. Lai atrastu ātrumu kādu jāpiešķir objektam, lai tas mainītu orbītu ja šis objekts atrodas uz Zemes:

1. Aprēķini (pēc enerģijas saglabāšanās orbītas laikā un, zinot orbītas parametrus, kā piemēram lielās pusass garumu - a) kādu ātrumu nepieciešams iegūt, lai pārvietotos attiecīgajā orbītā
2. Aprēķini kāds šis ātrums ir attiecībā pret Zemes ātrumu.
3. Aprēķini (pēc enerģijas) kāds ātrums jāpiešķir objektam, lai sasniegtu šo ātrumu **un pārvarētu Zemes gravitācijas ietekmi**.

**NB** Ļoti bieži pieļauta kļūda šādos uzdevumos ir lielās pusass definīcijas sajaukšana – tā ir **puse** no attāluma starp elipses vistālākajiem punktiem.

## Coriolis Force:

In physics, the Coriolis force is an inertial or fictitious force that acts on objects in motion within a frame of reference that rotates with respect to an inertial frame.

, where – angular velocity of the frame’s rotation (with a obtained from the corkscrew rule). From this equation we can conclude that if an object moves parallel to the axis of rotation of the frame, it experiences no Coriolis force, but if it moves perpendicular to the axis of rotation it experiences maximum Coriolis force.

# Electric Circuits:

**Termini:**

*Current source -* a component whose job is to provide a constant amount of current, outputting as much or as little voltage necessary to maintain that constant current.

# Thermodynamics

## Ideālās gazes

Pamata noderīgie vienādojumi:

The *equipartition of energy* theorem states that every degree of freedom of a molecule has an energy per molecule. ( per mole), since: (thus is the constant for individual molecules, but is the constant for moles of a substance.)

If f is the number of degrees of freedom then the internal energy of a gas is:

And from this we can derive the molar specific heat (siltuma daudzums, ko viens mols vielas uzņem vai atdod, sasilstot vai atdziestot par 1 K grādu) of a gas at constant volume or constant pressure:

Also:

Also from the *equipartition of energy* theorem since translational motion has 3 degrees of freedom the average translational kinetic energy per molecule of an ideal gas is:

We can derive the formula for the root mean square speed from this:

, where – the molar mass

And we can also work out the average value of the speed along one axis squared:

## Ideal gas Law:

Work of gas:

* This means that, when given a graph of the ideal gas you can obtain the work done in a particular instance as the area under the curve. The total work done by the gas during a closed loop process is the area enclosed by the process’ curve.

More important considerations about internal energy of a gas:

And if two containers of the same gas have the same temperature and amount of molecules, their internal energies are equal since

# Uzdevumi par lietderības koeficientu, vai pievadīto siltumu:

Galvenā lieta, ko šajos uzdevumos atcerēties ir, ka **pievadītais** siltums NAV vienāds ar gāzes padarīto darbu closed loop procesos. Kopējais siltums closed loop procesā ir **pievadītais – aizvadītais** siltums.

**Polytropic processes:**

A polytropic process is a thermodynamic process that obeys the relation:

where p is the pressure, V is volume, k is the polytropic index. The polytropic process equation can describe multiple expansion and compression processes which include heat transfer.

Some specific values of n correspond to particular cases:

k=0 for an isobaric process,

k=∞ for an isochoric process.

In addition, when the ideal gas law applies:

k=1 for an isothermal process,

Graphical user interface

Description automatically generated with low confidencek= for an isentropic process.

## Additional facts

For molecule velocities that are much smaller than the thermal speed the Maxwell distribution is almost constant (the velocity components of the molecules are evenly distributed from -to)

## Boltzmann Law

The Boltzmann Law states that

While simple to state, it has profound implications.

Examples of usage:

* **Atmospheric pressure** - The pressure in the atmosphere at height h is proportional to the probability that a molecule will be at that height, and is therefore (where we have assumed constant T and taken E to be the gravitational potential energy of the molecule). Also (for constant T atmospheres) .
* Probability that a molecule in the air will have the x-component of its velocity equal to .
* **Vapour pressure -** The probability that a water molecule in a mug of tea has enough (or more than enough) energy to leave the liquid is proportional .

**Gas in motion:**

In the case of gas flow **along straight streamlines** momentum conservation gives:

i.e. if a gas starts from zero velocity, and changes its speed due to pressure differences

# Air moisture:

There are two processes going on at the liquid/gas interface: condensation and evaporation.(The evaporation rate increases very rapidly with temperature. Condensation rate, on the other hand, is less sensitive to temperature) Obviously, if the evaporation rate exceeds the condensation rate, the amount of liquid is decreasing, and vice versa.

If we take a certain amount of liquid and seal it tightly into a bottle, the two processes reach an equilibrium: the concentration of the vapour molecules in the gaseous phase will reach such a value that the evaporation rate equals the condensation rate.

**Saturation vapour -** Vapour with such a concentration which leads to the condensation rate being equal to the evaporation rate at the given temperature

The amount of vapours is measured as the partial pressure caused by the given type of molecules in the gaseous phase. (Valid owing to Dalton’s Law which states that the total pressure exerted by a gas equals to the sum of partial pressures). Thus, the saturation vapour is typically characterised via its pressure, **the saturation vapour pressure.**

From there we obtain the definition of relative humidity.

**Relative humidity** - is defined as the ratio of the vapour pressure to the saturation vapour pressure at the given temperature of the gas

Diagram

Description automatically generated with low confidence, kur p – ūdens tvaika parciālspiediens un p0 – piesātināta tvaika parciālspiediens tajā pašā temperatūrā.

**Facts:**

* Liquid will start boiling if the condition is satisfied.
* If we have an interface between 2 liquids, bubbles at the interface can be entered by molecules of both liquids in the vapour phase. Taking in to account Dalton’s law we can conclude that At the interface of two liquids, boiling can start at considerably lower temperatures than in both liquids, separately: boiling will start when the condition is satisfied.

## Surface tension:

The molecules of a substance in the liquid phase are being attracted by the other liquid molecules and therefore have a certain negative potential energy with respect to infinity.

As compared with the molecules in the bulk of the liquid, the number of attracting neighbours is smaller for the molecules directly at the surface and thus, the negative potential energy is also smaller by modulus.

This missing negative energy can be interpreted as a positive energy of the surface - proportional to the number of molecules at the surface, which is in its turn proportional to the surface area of the liquid,

, where the coefficient of proportionality is called the surface tension.

Similarly to the tension in rope, we can say that if we make an imaginary cut line of length L on the surface, the two halves of the surface pull each with force

## Capillary pressure:

Now, let us study the capillary pressure which is the gauge pressure due to spherical liquid-air interface of radius , such as one would have in the case of a bubble inside a liquid. Capillary pressure is essentially pressure resulting from the surface tension forces.

The gauge pressure due to capillary forces across a **curved interface** is

in spherical geometry, and

in cylindrical geometry.

These expressions can be generalized to arbitrary shapes of the interface:

, where and are the curvature radii of two curves at their crossing point P.

# Electric Circuits

## Thenvin’s theorem

Diagram, schematic

Description automatically generatedAny circuit which consists only of resistors and batteries and has two ports A and B can be modeled as a series connection of a battery and a resistance.

Since the behaviour of the original and substitution circuits must be identical, we can find the equivalent electromotive force and resistance via these rules:

1. The equivalent can be found as the voltage difference between the leads A and B, when there is no external load connected between these leads
2. The equivalent can be found as the resistance between the leads A and B when all the ideal internal electromotive forces are substituted by wires. (*this is because the original and the substitution circuits must have identical increase of the lead voltage when there is a certain increase of the lead current, and an ideal battery and a piece of wire have identically a zero voltage response to an increase of the current*)

# Method of Complex Impedances

The Method of Complex Impedances can be used to analyse AC circuits.

**Refresher of Complex number properties and operations:**

**Forms:**

Any complex number can be written in following forms:

**Polar form:**

**Regular form:**

, where

and

**Interpretation of polar form multiplication:**

If we use the polar form of a complex number, where we essentially can interpret the complex number as a phasor, then multiplying the complex number (phasor) by another complex number

we see that the mathematical operation of multiplying essentially means incrmeenting the phase of the phasor by the angle .

**Interpretation of polar form summation:**

If we model two compex numbers in their polar form as phasors, the sum of these two complex numbers would be the sum of the two phasor vectors (because we would just essentially sum the x and y components seperately).

**Inverse Euler:**

The inverse Euler formulas give us the sines and cosines in terms of complex exponentials.

**Mathematical representation:**

Within this method we decsribe the oscillating voltages and currents using complex numbers. We use the following representations:

, where is a complex number and **our representation of voltage** – essentially a phasor that is rotating around with angular frequency .

, where is a complex number and **our representation of current** – essentially a phasor that is rotating around with angular frequency .

The key, however, of this method is to use impedances to relate the complex voltages and currents (just as we would with DC currents). Every circuit element produces a specific relation between the voltage and current, according to the equation:

I.e. the impedances decribe how the element react to voltage and they are the following:

We can write down the relation for the entire circuit or any subpart, just as we would for a network containing only resistors, and thus determine the relations between our representations:

**Geometrical representation:**

As said previously, there representation can be interpreted as phasor rotating around the complex plane.

Diagram

Description automatically generatedThe and vectors rotate around in the complex plane with the same angular speed ω. **The horizontal projections (the real parts) are the actual quantities that exist in the real world**. Since the vectors keep the same rigid shape with respect to each other, it follows that if the complex voltages and currents satisfy Kirchhoff’s rules at a given time, the actual voltages and currents satisfy Kirchhoff’s rules at all times.

# Power Calculation

Animportant case is the power dissipated in the circuit: this is anonlinear function of the voltage and current, and so we needto be careful.

To calculate the power dissipation we can use the following relations:

, where – the phase difference between the voltage and current

, where – is the complex conjugate of our current representation

# Summary and method guide:

The general procedure for using this method is the following:

1. Assign impedances of R , iωL, and 1 / iωC to the resistors, inductors, and capacitors in the circuit, and then use the standard rules for adding impedances in series and in parallel
2. Write down = Z for the entire circuit or any subpart, just as you would for a network containing only resistors.
3. Express the searched for quanitity, for example , in the form , and then express the polar form of using the properties of complex numbers i.e. and
4. Then finally, if necessary, we can express the actual voltage or current as the real part of the time-dependant phasor representation I = =

Remember that throughout this method, all Kirchoff’s rules apply and we can sum and subtract all currents and voltages accordingly. (Remember: “if the complex voltages and currents satisfy Kirchhoff’s rules at a given time, the actual voltages and currents satisfy Kirchhoff’s rules at all times.”)

A full overview of this method and example problems can be found in Purcell and Morin *Electricity and Magnetism* Chapter 8.

# Diferenciālvienādojumi

Vienmēr atrisinājums ir formā:

Attiecīgi var izmantot šo formu, lai izteiktu meklēto funkciju (pielīdzinātu) un iegūtu sakarības. Tādejādi iegūstot vispārīgo atrisinājumu. Ja atrisinājumi ir vairāki, tad var tos saskaitīt, lai paplašinātu “brīvības pakāpes” un tad šīs funkcijas koeficientus pielāgot atkarībā no sākuma stāvokļiem, izmantojot informāciju par funkciju un tās atvasinājumiem.

# Optics

## Lenses

When working with lenses one of the main equations we have to know is the lens maker’s equation:

where – index of refraction for the lens and – the index of refraction for the environment. Often times in Olympiads, more complex lens systems will be given. For this we can use the idea of summing up the optical power of different lenses that are next to each other:

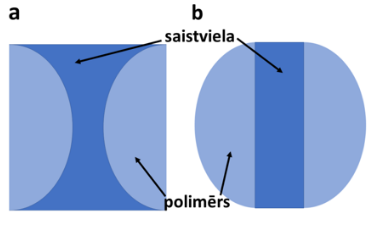
(see pr. 6. NBPhO 2022)

A picture containing icon

Description automatically generatedA picture containing shape

Description automatically generatedThis idea becomes very powerful in unison with the **divide and conquer** method. For more complex lens systems with different indexes of refraction, we can split the lenses up and treat them as different lenses:

We can then use the lens maker’s equation to get the optical strength of the individual lenses and thus the optical strength of the lens system.

Another visual example:

Note: if the lens systems are in a different environment than air, we have to be more careful. We can still deconstruct the lenses into various half lenses and so on, but we need to take into account the different indexes of refraction for the environment in our lens maker’s equations for all the deconstructed lenses.

In the cases where the two lenses we wish to combine have a distance between them which is too large to ignore, but still small enough for the system to be treated as one lens, we can use the following formula to determine the net optical power (see Upgrade Your Physics 5.2.3.3 for full derivation)

A picture containing line, diagram, parallel

Description automatically generatedAnother useful relation (which in fact can be used to derive the relations for thin lenses and is obtained via the small angle approximation) is:

## Obtaining images:

Next it’s the two main equations for obtaining an image from info on the lens and object:

We can find the size (with the magnification) of any image and the location of any image with these equations. From then on we can treat the images as if they were real objects we were looking at. (Exceptions might, however, occur, for example if the viewer is in a weird place like if the image forms behind the viewer)

## Rays:

Really, almost all geometrical optics equations are obtained by simple geometrical arguments like *similar triangles.* When in doubt, it might be a good idea to return to the basics and just trace the movement of light rays, and think about the geometry of them.

Ray properties:

* Rays going through the center of a lens don’t change direction
* Rays going parallel to the general optical axis cross the focal point
* Rays parallel to a secondary optical axis (any axis through the center of the lens) cross at a point on the focal plane

A picture containing diagram

Description automatically generatedŅūtona formula:

## Other advice on optics

* To distinguish something/see clearly usually means that the certain object is located at the focus or the image is formed at the focus.

# Electromagnetism

**Method of images**

Boundary value problems or problems involving boundaries can often be solved by the method of images.

<https://en.wikipedia.org/wiki/Method_of_images#:~:text=The%20method%20of%20images%20(or,respect%20to%20a%20symmetry%20hyperplane>.

It can be used in electrostatics to simply calculate or visualize the distribution of the electric field of a charge in the vicinity of a conducting surface (see Izlases kursi), or

It can be used Magnet-superconductor systems (see pr. 3. EuPhO 2017)

Or to model different flows or reflections at a boundary.

## Symmetry between electric and magnetic fields:

There exists an interesting symmetry between the equations that govern electric and magnetic fields. Most equations of electric and magnetic fields are analogous to each other, and can be derived one form the other using the analogy that

In this way one only has to remember one equation (usually for the electric field) and can derive the other one from analogy.

**Coulomb Law and Biot-Savart Law (differential form):**

**Field of Electric and magnetic dipole:**

, where , and

**Electric and magnetic field energy density:**

**Magnetic monopole method for magnetic dipoles**

The aforementioned symmetry can also be used in a kind of practical sense.

When working with magnetic dipoles we can model their interaction with other objects (including other dipoles for that matter) by using magnetic monopoles.

A magnetic dipole can be modelled as a pair of magnetic monopoles of strength qm and −qm separated by a distance d such that = qmd.

Then we can use Maxwell’s Law’s for electric monopoles (charges) analogously on magnetic monopoles, including Gauss’ Law (thus by using the binomial expansion deriving the magnetic field a distance x>>d away from the dipole) and also use that the force is −qmB(x) + qmB(x + d).

# Special Relativity:

One symmetrical form of Lorentz transformations that helps to remember the equations:

## Four Vectors

The Lorentz transforms show you how to work out the relationships between the (t,x,y,z) co-ordinates measured in different frames of reference. We describe anything that transforms in the same way as a four vector, although strictly speaking we use (ct,x,y,z) so that all the components of the vector have the same units. Three other examples of four vectors are:

is called the four velocity of an object, and is the derivative of (ct, x,y,z) with respect to the proper time τ. Proper time is the time elapsed as measured in the rest frame of the object t=γτ.

the momentum four vector. Here m is equal to . This must be a four vector since it is equal to the rest mass multiplied by the four velocity (which we already know to be a four vector).

It also turns out that the dot product of **any** two four-vectors is ‘frame invariant’ – in other words all observers will agree on its value. The dot product of two four-vectors is slightly different to the conventional dot product, as shown below:

Notice that we subtract the product of the first elements.

The dot product of the position four vector with the wave four vector gives

Especially useful one often finds the conservation of the dot product of the momentum four vector. (example: pr 1. NBPhO 2021)

# Approximations

**Taylor Series**

# Mathematics for physics

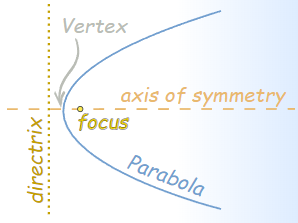
**Average value of function in given interval a-b:**

# Geometry for physics

**Parabola**

A picture containing chart

Description automatically generated**Definition:** A parabola is a curve where any point is at an equal distance from ***the focus*** of the parabola and a fixed straight line (***the directrix***).

***The vertex*** is halfway between the focus and directrix.

**Chart, line chart

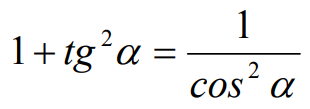
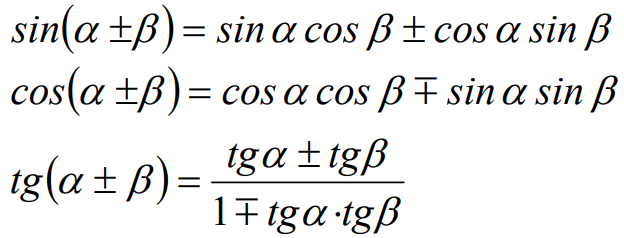
Description automatically generatedReflection:** Any ray parallel to the axis of symmetry gets reflected off the parabola straight to the focus.

Consequently the angle between the parallel ray and the tangent to the parabola at a point P, is equal to the angle between a line connecting the point P, and the tangent of the parabola at point P. (see fig. above)

**Sketchy property from EuPhO 2019:** for any point on a parabola, its distance from the focus plus its distance from a horizontal line is constant.

**Ellipses**

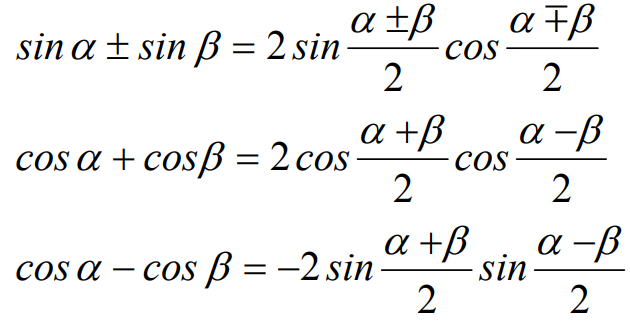
**Trigonometry**

**1**

**2**

**1** can be derived from relation of cosine and sine, and **4** can be derived from **2** and **3**

**3**

**4**

You really only have to remember the postive (i.e. summation) parts of **2, 3 and 5** as the negative (i.e. subtraction) parts can be derived from the positive

**5**

**6**

**6** is essentially the same as **5** only with a *cos* in the front

**7**

**7** is **6** with *cos* replaced with *sin* and a minus sign in front